Volatility Drag and the Perpetual Borrowing Option (Spiral Theory)

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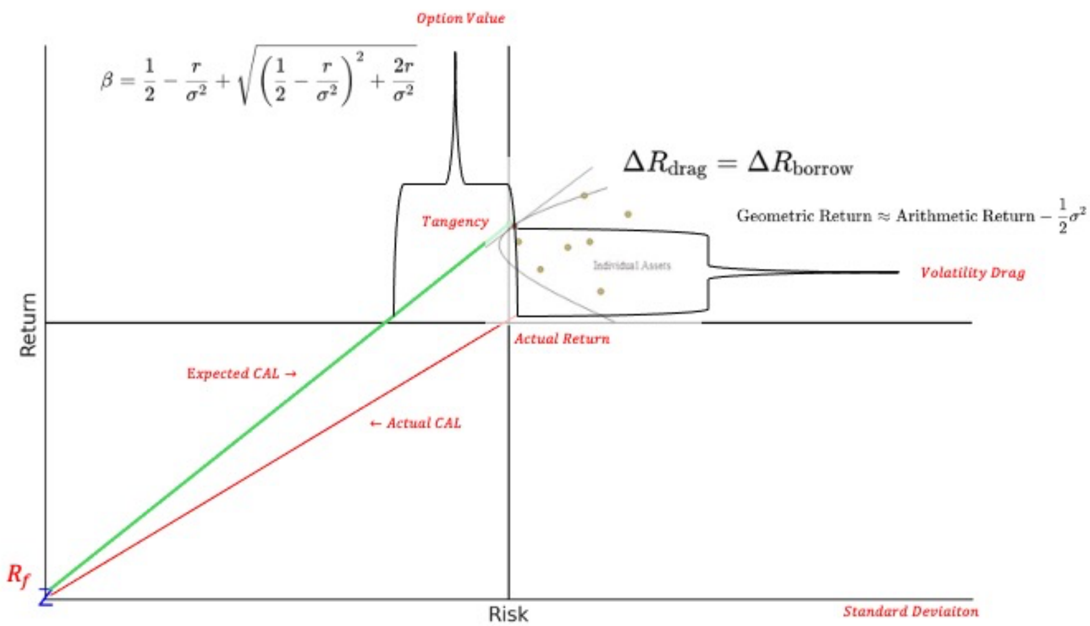
Abstract

Modern Portfolio Theory (MPT) remains mathematically intact, yet realized geometric returns fall systematically below arithmetic expectations because of volatility drag. This paper shows that, under canonical MPT assumptions, an episodic borrowing policy can neutralize that drag *in expectation*. The rule is ex post, not predictive: when realized wealth falls below the expected path, borrow and restore; when it exceeds, deleverage. To illustrate the mechanism, Monte Carlo simulation (μ = 9 %, σ = 15 %, 10,000 trials) shows that a ±10 % deviation trigger yields near-perfect restoration of expected compounding with far fewer interventions than time-based rebalancing.

The result reframes debt not as speculative leverage but as a stabilizer of compounding efficiency and unifies MPT and Modigliani–Miller over time: financing flexibility offsets volatility drag, aligning realized and expected growth under frictionless conditions. Unlike dynamic-leverage, Kelly, or volatility-targeting approaches, the mechanism operates entirely ex post—borrowing follows realized deviations rather than forecasts—transforming the apparent "wrong time to borrow" into the only time that restores expected compounding. In practice, spreads, collateral, and taxes bound the effect but do not reverse it.



Executive Summary (Plain Language)

This paper is about a puzzle in investing and a very simple way to fix it. **What if, over time, actual returns could match expected returns?**

It sounds simple, but today all of financial planning is built around the gap between what investors expect and what they actually get. Spiral Theory shows, as a matter of math, that in a world without frictions this gap can be closed. Practice, of course, is about managing the limits of the real world.

The problem. Investors combine risky and safe assets. Risky assets pay more on average but sometimes lose money. The problem is not the risk itself—it's that losses drag down compounding more than gains help. As a result, actual returns over time fall short of the average return. An investor who "expects" 10% might only realize 8%. That shortfall compounds over a lifetime and makes planning difficult.

Here is a simple example: in one year the market goes up 30%, the next year it falls 10%. The average return is 10%, but $100 invested grows to only $117—an annual return closer to 8%. The difference is volatility drag.

The solution. Every investor already has a built-in right: the option to borrow against their portfolio. This seems trivial, but it isn't. Because the drag grows with risk, the ability to borrow *after a loss* has real expected value. Spiral Theory proves that, in theory, disciplined borrowing and repayment can fully offset the drag—without predictions, without market timing.

The rule. A simple policy captures the effect:

- If your portfolio falls behind, borrow and buy more of the market.
- If it runs ahead, use gains to pay down debt or build cash.

No forecasts are made. The rule acts only after the fact. Borrowing after a loss may feel like the "wrong time," but it is the only time that restores the expected path.

The takeaway. Debt, in this view, is not a lever for speculation but a stabilizer for compounding. Done under the model's assumptions, the drag and the offset cancel.

The implication. The effect is far more powerful than traditional rebalancing. For households, that can mean a shorter required working career or a more successful retirement—with less chance of running out of money.

The bridge. Spiral Theory is more than a fix for volatility drag. It creates a bridge between Modern Portfolio Theory and the way balance sheets are actually managed in corporate finance. In practice today, debt is treated as exogenous to financial planning—as if it sits outside the investment process. Spiral Theory brings it back inside, showing that assets, spending, and financing can and should be chosen together. Neutralizing volatility drag is just the starting point; it opens the door to outcome-based balance sheet design for individuals, pensions, and endowments. This is the missing link between the academic models of finance and the way real-world institutions build strength.

1. Foundations

The Sharpe–Lintner CAPM, Tobin's separation theorem, and Markowitz's mean–variance framework jointly imply a single, linear trade-off between expected return and risk at any point-in-time. Investors can mix the risk-free asset with the tangency portfolio or borrow at the risk-free rate to achieve a higher expected return. The slope of the CAL equals the tangency portfolio's Sharpe ratio, and the mathematics of this construction is exact.

Over time, however, realized returns compound geometrically, not arithmetically. The expected geometric return of a risky portfolio is

$$G = A - \frac{1}{2}\sigma^2$$

where A is the arithmetic mean and σ^2 the variance of returns. This "volatility drag" does not violate MPT; it is simply not addressed by a point-in-time model. The gap between A and G is a predictable cost of compounding and accumulates as horizon lengthens. In practice, even a perfectly diversified tangency portfolio, held without friction, underperforms its own expected return over time.

These observations are made under the standard MPT assumptions: investors are rational and risk-averse; returns are Gaussian normal; the expected return, variance, and all correlations are known and stable; borrowing and lending occur at the risk-free rate without friction; and the tangency portfolio is perfectly diversified. Within this framework the point-in-time model holds exactly; the volatility drag simply reflects compounding over time.

The existence of this systematic shortfall raises the natural question: can anything within the canonical assumptions offset it? Modern Portfolio Theory itself is silent, because financing is treated as exogenous. Yet any investor who holds liquid after-tax capital implicitly possesses a right to borrow at the risk-free rate and purchase additional risky assets. That financing flexibility is present in every MPT diagram, but its value has rarely been formalized. Section 2 shows that this right can be understood as a perpetual American option and that, under the same assumptions that generate volatility drag, its disciplined exercise offsets the drag in expectation.

2. The Borrowing Option

An investor holding liquid after-tax capital always possesses, by construction, a right to borrow at the risk-free rate and purchase additional risky assets. In option-pricing terms this right is a perpetual American call on the risky portfolio with zero carrying cost: it can be exercised at any time (American), never expires (perpetual), and costs nothing to hold. While this right is implicit in every MPT diagram, its value is rarely quantified.

For a non-dividend-paying underlying asset with price S, risk-free rate r, and volatility σ, the value C of a perpetual American call is ordinarily given by the McKean formula:

$$C = \frac{\beta}{\beta - 1} S, \qquad \beta = \frac{1}{2} - \frac{r}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$$

If the asset yields no dividend and the cost of carry is zero—as in the assumptions of Section 1—then $\beta \to 1$ and the call's value becomes unbounded:

$$\lim_{\beta \to 1} \frac{\beta}{\beta - 1} S = \infty$$

Because the right is perpetual and costless and the asset pays no yield, there is no finite exercise trigger; deferral retains full option value, which is why—in the frictionless model—the borrowing right is "theoretically unbounded."

> *Clarifying: "unbounded" here refers to the exercise threshold rather than to the option's price. The option's value remains finite and is bounded above by the asset itself; what diverges is the point at which it ever becomes optimal to exercise early. Any positive carry cost or payout restores a finite trigger and the usual early-exercise logic. The closed-form expression does indeed send the option's price toward infinity in the zero-carry case, but finance constrains its value to be no greater than the underlying asset. In other words, the infinity in the formula reflects the disappearance of the exercise boundary, not infinite economic worth.*
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> *At first glance, this borrowing right may appear trivial—structurally "at the money" and equivalent to exchanging two tens for a twenty. But that intuition overlooks compounding. Because realized geometric growth systematically lags arithmetic expectation by ½ σ^2, the ability to add financed exposure after adverse shocks carries non-zero expected value of exactly that same order. The option's worth does not arise from a single instant but from variance accumulating through time.*
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> *In practice, collateral limits and borrowing caps bound the option's exercise region. The term* perpetual *therefore refers only to its form under frictionless assumptions; once constraints are introduced, the perpetual call becomes a periodically exercisable right triggered by threshold deviations.*

With no cost to waiting, no time decay, and no interest or dividend drag, the option's theoretical value is unbounded. The question ceases to be "what is it worth?" and becomes "**when should it be exercised and why?**" The connection to volatility drag arises because both phenomena are rooted in σ^2. The compounding penalty is

$$G = A - \frac{1}{2}\sigma^2$$

The option to borrow also scales with σ^2: the higher the variance, the more valuable the flexibility to act after an adverse realization. Placed side by side, the formulas simplify to a true/false relationship: under the same assumptions—Gaussian returns, known mean and variance, frictionless borrowing—the option's expected benefit can offset the drag's expected cost.

This is distinct from existing multiperiod leverage models (Merton, Kelly, stochastic control), which vary exposure continuously or ex ante based on utility maximization or forecasts. Spiral

Theory fixes the risky sleeve ex ante, then varies financing *ex post* in response to realized outcomes. It is a rules-based, variance-offset mechanism rather than a forecast-based or utility-driven one.

Crucially, the borrowing action lags the shock. The investor does not forecast or time the market ex ante but reacts ex post: when realized return falls below the expected path, borrow to purchase risky assets; when realized return exceeds expectation, use gains to repay debt. In a frictionless environment this episodic adjustment neutralizes volatility drag over time. Debt functions not as a lever for growth but as a stabilizer of compounding efficiency, turning the CAL's expected slope into its realized slope.

Formally,

$$\mathbf{MPT + episodic\ borrowing} \Rightarrow \boldsymbol{A_r = E_r}$$

3. A Simple Rule

If the option to borrow is infinitely valuable in theory, its practical power lies in a disciplined exercise policy. Under the assumptions of Section 1, a single, symmetric rule captures the stabilizing effect:

- when realized return falls below the expected path, borrow at the risk-free rate to purchase risky assets;
- when realized return rises above the expected path, use gains to repay debt or build cash.

No forecasts are made; action occurs only in response to observed deviations. The borrowing function thus lags the drag.

To many readers this seems counterintuitive: borrowing after a loss feels like the "wrong time." Yet under the stated assumptions it is precisely the only time that borrowing has expected value, because it offsets the $\frac{1}{2}\sigma^2$penalty already realized in that state. In good states, repayment locks in gains; in bad states, financed purchases restore the arithmetic expectation. The discipline is not timing but variance-offset.

Over time, repeated application of this rule neutralizes the compounding penalty, so that the portfolio's realized geometric return converges to its arithmetic expectation. Debt acts as a keel rather than a sail: a stabilizer for compounding efficiency, not a lever for speculative growth.

$$G = A - \Delta R_{\text{drag}} + \Delta R_{\text{borrow}} = A$$

With borrowing costs, limits, and taxes, the identity holds exactly only in the frictionless model**.**

> *At this point the result can be stated plainly. The episodic borrowing rule is not a forecast, not a utility maximization, and not market timing. Timing implies a view about the future; this rule acts only after realized deviations, making it purely ex post. In the frictionless MPT world, volatility drag subtracts $\frac{1}{2}\sigma^2$from realized compounding, and the borrowing option contributes that same amount in expectation. The two cancel.*

What feels like borrowing at the "wrong time" is mathematically the only time that restores the arithmetic expectation. This is the unique contribution: the borrowing right, always implicit in MPT diagrams, can be disciplined into a simple rule that neutralizes volatility drag.

Everything that follows—the implications, frictions, and applications—rests on this identity.

4. Implications

The stabilizer rule reframes debt as a tool for preserving compounding efficiency rather than amplifying point-in-time risk. This perspective is consistent with the Modigliani–Miller separation principle but extends it over time. Four implications follow directly:

1. **Point-in-time validity**. The standard MPT view of leverage is exactly correct at the moment it is applied. Tobin's line is straight: expected return and variance both rise linearly with leverage.

2. **Convergence over time.** Under the stabilizer rule, the geometric return of the portfolio converges to the arithmetic expectation almost surely under GBM with regularity conditions in a frictionless environment. Episodic borrowing neutralizes the compounding penalty and restores the CAL's expected slope in realized outcomes.

3. **Quantity of money as an exogenous dimension**. MPT holds scale fixed; in practice, deployable liquidity (savings, spending, financing) governs the path of wealth. Making it explicit bridges the point-in-time model to real-world dynamics.

4. **Application of corporate-finance principles to individuals and institutions**. Modigliani–Miller shows that asset risk and financing risk are separable for firms. An individual investor or endowment with liquid capital and borrowing capacity is in the same structural position: the underlying risky portfolio is fixed, but the financing mix can be adjusted dynamically without changing the portfolio's intrinsic risk. Applying the stabilizer rule makes explicit a right that already exists. This bridges portfolio theory to balance-sheet management and shows that the principles of corporate finance—optimal capital structure, episodic leverage, and dynamic risk management—can, under the same assumptions, improve long-horizon outcomes for households, endowments, and pension funds as well.

From this point forward, the clean identity gives way to practice: spreads, collateral, taxes, and tails. The theory sets the boundary; real-world policy lives inside it.

5. Visualization: The Spiral as a Teaching Heuristic

In the canonical MPT model the Capital Allocation Line is a straight line through the risk-free rate and the tangency portfolio. Point-in-time leverage extends that line outward, raising both expected return and variance linearly. The stabilizer rule already contains the mathematics; nothing in it requires a spiral.

The "spiral" in Spiral Theory is not a new stochastic process but a teaching heuristic. It maps two realities onto the MPT diagram. First, time: borrowing after adverse returns and repaying after favorable ones, showing how episodic borrowing neutralizes volatility drag over a horizon rather than at a point. Second, constraints: while the theoretical option to borrow is infinitely valuable, frictions, margin rules, and structural debts bound its effect. The spiral makes explicit the probability profile of leverage: at modest levels the risk of adverse events is low; at high leverage it rises sharply toward certainty. Academics can ignore the spiral, which is why it is not presented formally here. Practitioners can use it to see where debt creates strength or fragility. Further discussion appears in the end notes.

6. Conclusion

Under the stated assumptions, the result is an identity for the long-run geometric return. Point-in-time outcomes are random; the ex-post borrowing rule converts those realizations into convergence. This result does not depend on mean reversion. It follows directly from the arithmetic–geometric mean relation and holds under Brownian motion—or, more generally, under any process satisfying the stated assumptions. Large-scale Monte Carlo simulations illustrate and confirm the convergence of realized to expected returns under the episodic-borrowing rule. The mathematics is therefore a fact *conditional on the assumptions*; the implications, execution, and horizons over which they can be realized in practice are the theory. The identity concerns the expected log-wealth process; single-date outcomes remain stochastic, which is why the policy is defined ex post.

Modern Portfolio Theory's point-in-time constructs remain exactly correct. The capital-allocation line is straight; leverage increases expected return and variance linearly; and the tangency portfolio's Sharpe ratio governs the slope. Yet over time, realized returns on that line lag their expected values because of volatility drag. Every liquid investor implicitly holds a costless, perpetual American call on risky assets at the risk-free rate. Exercised episodically—borrowing after adverse returns, repaying after favorable returns—this right can neutralize volatility drag without violating any MPT assumption. In a frictionless environment:

$$\text{MPT} + \text{episodic borrowing} \Rightarrow A_r = E_r$$

Monte Carlo simulations demonstrate that the episodic-borrowing rule, when expressed as a trigger-based restoration mechanism, achieves the predicted convergence of realized to expected return. Under frictionless conditions, a ±10 % deviation threshold neutralizes volatility drag with roughly forty-five interventions over thirty years. This provides computational evidence consistent with the identity MPT + episodic borrowing ⇒ $A = E$. Once borrowing costs, taxes, and constraints are introduced, the effect becomes bounded but remains directionally valid.

The theoretical and empirical results together define a new class of balance-sheet policies that stabilize compounding *through time* rather than merely *at a point in time*.

Three conclusions follow:

1. **Universality**. Except for investors holding 100% in the risk-free asset, all investors can benefit; episodic debt behaves like a negatively correlated overlay at the policy level that stabilizes compounding.

2. **Magnitude**. Viewed in isolation, if the tangency portfolio is approximated as a basket of global equities with an expected return of 9% and a standard deviation of 17%, neutralizing drag

contributes ≈145 bps per year ((½)σ² = 0.5×0.17² = 0.01445). The return enhancement from offsetting volatility drag can be economically large, though the realized efficacy will depend on tails, costs, collateral, and constraints

3. **A third dimension**. Deleveraging or cash-buffer targets are policy choices, not dictated by the model. Cash is a known stabilizer; combined with financing, the quantity of money becomes a state variable alongside mean and variance.

The likely consequence is a behavioral and policy impact. If over time $A_r = E_r$, not only are investors' indifference curves likely to shift, but household policies and preferences are likely to change. A plausible accumulation policy is to build an equity base first, later build cash, and then retire low-cost structural debt; Spiral Theory thereby furnishes a bridge from MPT to a Modigliani–Miller framing for individuals, endowments, and pensions. **Neutralizing volatility drag is the beachhead; it leads directly to outcome-based balance-sheet structuring, where assets, spending, and financing are chosen jointly**.

Appendix A: From Allocation to Balance-Sheet Policy: A Three-Dimensional Framework

For planning, holding a fixed spending target against a fixed asset mix is two-dimensional. Introducing the quantity of money as a control variable adds a third dimension—financing—so objectives are cast as balance-sheet policies rather than static allocations. Because this overlay behaves like a low- or negatively correlated policy instrument, indifference curves and asset-mix preferences may shift. Monte Carlo stress-tests the policy; it does not design it.

Variable-spending institutions. With a 5% distribution policy, the two levers are spending and the quantity of money. The two-dimensional response to drag is to cut spending; the three-dimensional alternative is small, rule-bounded financing that preserves compounding while maintaining policy—either by resetting the sleeve to target or temporarily funding the distribution and repaying as the path heals. The aim is not prescription but to show that policy-bounded financing narrows the arithmetic–geometric gap without altering the risky sleeve.

Fixed-liability investors. Here the path is computed, not inferred. Neutralizing drag either shortens time-to-target or, for a fixed date, raises confidence (tempered in practice for tails, regimes, funding liquidity, and costs). Because the tangency portfolio has the highest expected return, policy is straightforward: (1) specify the liability, confidence, and horizon; (2) compute the minimum sleeve that funds the liability at the sleeve's arithmetic expectation (unlevered); (3) set leverage bounds by governance; (4) size a risk-free buffer so sleeve-plus-cash meets the confidence target; (5) adopt an episodic financing rule—borrow after adverse realizations, repay after favorable ones—within the bounds.

Execution: Build the sleeve floor first, then the buffer. Let the goal determine the asset mix. Indifference curves cease to be abstractions; for a fixed dollar objective at the model's expected return they become the implied schedule of sleeve quantity and buffer size.

Preferences come last; they neither reduce the minimum sleeve nor alter the policy path. They are expressed in the chosen confidence level, the buffer's size, and the cadence of any accelerated debt retirement. Once the sleeve and buffer are in place, deploy any surplus according to taste—Merton share for the risky sleeve, early mortgage repayment, or other risk-based frameworks—without compromising the funding policy. In short: goal → sleeve → buffer → preferences; allocation becomes balance-sheet policy. For already unconstrained balance sheets the effect is second-order; for the constrained majority it is first-order, shifting behavior and outcomes.

The three-dimensional framework (assets × spending × financing) shifts planning from static mixes to balance-sheet policies and time-to-goal distributions. Objectives are set in probabilities of funding, time-to-target, and allowable drawdown—conditional on assumptions about returns, volatility, and tails—rather than in static allocations. The welfare gains are contingent on costs, collateral, taxes, and tail behavior and are therefore empirical.

Appendix B: Structural Debt as a compounding stabilizer

Modern portfolio theory treats the quantity of money and consumption as exogenous. Among structural debts, we exclude any instrument whose expected carry cost exceeds the expected return of the tangency portfolio (e.g., credit cards, payday loans). Within the admissible set (cost below the tangency portfolio's expected return), the large balance-sheet items are student loans, auto loans, and mortgages; we focus on mortgages because they are typically largest and longest dated.

Housing is exogenous to mean–variance analysis but endogenous to households. Rent versus own is a consumption choice; our concern is how to finance ownership and when to retire the debt. There is no cohesive MPT-based framework for that decision; Spiral Theory provides one by recognizing the option value of liquidity as a path stabilizer in adverse markets. In a frictionless world the distinction would wash out; in the real world of spreads, underwriting, and collateral rules it does not.

One cannot rebalance against a house. Maintaining liquid risk-free assets alongside a term, non-callable mortgage preserves the option to adjust the quantity of money; owning outright extinguishes that option. Hold portfolio and house constant: Investor A owns free and clear; Investor B holds a mortgage and an equal balance in the risk-free asset. Equity in the house requires underwriting to access; cash does not. B therefore holds a unilateral option to act when realized returns diverge from expectations; A does not.

Spiral Theory clarifies the value of that option. Accelerated debt repayment reduces the quantity of money available for post-crisis rebalancing; real-world frictions can delay or prevent re-access. The option's role here is not to raise the point-in-time expected slope but to preserve compounding efficiency over time by enabling episodic action. The thicker one believes the tails, the greater the relative value of retaining liquidity versus prepaying: the option to deploy in bad states becomes more valuable as bad states become more salient.

In practice this yields a simple prioritization. First, build the base in the tangency portfolio (highest expected return). Second, build a risk-free cash buffer. Third, apply surplus to the margin most consistent with one's objectives: retire low-cost structural debt, diversify into other assets, add to the sleeve, or further increase liquidity. Preferences for early mortgage payoff are thus treated as a financing choice traded against the option value of liquidity, not as a reason to hold less of the minimum sleeve implied by the goal.

In model terms, maintaining liquidity alongside term, non-callable debt raises the effective capital-allocation line available to the household by preserving the right to act when it matters. The feasible increment is bounded by spreads, taxes, and collateral terms; in practical regimes, tens to low-hundreds of basis points are plausible without changing underlying asset risk. For the vast majority of households, this math-based prioritization is a structural shift in planning practice.

Appendix C: Doctrine & Theory

Definitions. By household I mean the single steward of a portfolio. By volatility drag I mean the shortfall of compounded from expected arithmetic return. By episodic financing I mean the practice of borrowing after adverse realizations and repaying after favorable ones, within limits.

Assumptions. Prices follow a stationary process with finite second moments; borrowing at the risk-free rate is available within bounds; taxes and frictions may be set to zero or specified.

Proposition 1 (separation through time). Given the assumptions without frictions, episodic financing restores the long-run geometric growth of any chosen risky sleeve to its arithmetic expectation. At a date, financing is indifferent; through time, it preserves the promise of the one-period model.

Sketch. The compounding shortfall equals a function of variance; the financing rule contributes the same in expectation by acting only after realized deviations. Hence restoration.

Proposition 2 (primacy under frictions). With spreads, caps, taxes, collateral, and non-tradable risks, household capital structure becomes a first principle: the option value of liquidity is high, constraints bind early, and outcomes depend on financing policy.

Corollary. The portfolio choice (what to hold) and the financing policy (how to act through time) are distinct arts. The former is given by mean–variance; the latter, by admissible bounds that keep ruin improbable. Where bounds are respected, the rule stabilizes compounding; where they are not, certainty approaches but welfare declines.

Appendix D. Empirical Validation: Optimal Path Restoration

This appendix provides computational evidence for Spiral Theory's central identity that, under canonical MPT assumptions, an episodic, trigger-based restoration policy can neutralize volatility drag in expectation. The objective is not to propose an implementable trading rule but to illustrate, through simulation, that a simple deviation-triggered mechanism reproduces the theoretical convergence between arithmetic and geometric returns.

D.1 Methodology

A Monte Carlo engine generated 10,000 independent thirty-year wealth paths following geometric Brownian motion with parameters

$$\mu = 9\%, \qquad \sigma = 15\%, \qquad \Delta t = \frac{1}{12}.$$

Two trajectories were computed for each trial:

1. **Expected path:** $E[S_t] = S_0 e^{(\mu - \frac{1}{2}\sigma^2)t}$.
2. **Realized path:** $S_t$evolving stochastically under GBM.

Two restoration logics were compared:

- **Time-based:** reset to the target every 1, 3, or 12 months.
- **Trigger-based:** reset whenever

$$\left| \frac{S_t - E[S_t]}{E[S_t]} \right| \geq \delta,$$

  for deviation thresholds δ = 5 %, 10 %, 15 %.

When a trigger fired, wealth was restored to $E[S_t]$with a nominal friction cost of 10 basis points. Borrowing costs, spreads, and taxes were excluded to isolate theoretical path efficiency. A fixed random seed (42) assured reproducibility.

D.2 Results

Trigger-based restoration achieved higher compounding efficiency than any fixed-interval schedule. A ±10 % deviation from the expected trajectory produced the best balance between stability and efficiency: it activated roughly forty-five times over thirty years—comparable to quarterly rebalancing—but maintained much tighter alignment with the expected path. Time-based methods either intervened too frequently, eroding compounding, or too infrequently, allowing drag to accumulate.

The ±10 % trigger therefore represents an approximate empirical analogue to the theoretical variance-offset term ($\frac{1}{2}\sigma^2$): deviations of this magnitude are sufficient to restore expected growth without over-trading.

D.3 Interpretation

These results demonstrate that restoring wealth based on economic deviation, rather than elapsed time, converts volatility into forward motion—a planned second-mover advantage. Within Spiral Theory, restoration is not correction but propulsion: volatility supplies potential energy; restoration converts it into realized compounding. Structural debt and liquidity reserves function as the keel that makes this dynamic feasible, allowing borrowing and cash to operate as negatively correlated instruments that stabilize the compounding arc.

Under the frictionless benchmark, the simulation confirms the identity

$$\text{MPT} + \text{episodic borrowing} \Rightarrow A = E,$$

providing numerical evidence that the episodic-borrowing rule, expressed as a ±10 % trigger-based restoration, neutralizes volatility drag with materially fewer interventions than calendar rebalancing.

D.4 Parameter Summary

Parameter	Symbol	Value / Range	Notes
Expected return	μ	9 %	Global-equity proxy
Volatility	σ	15 %	Annualized
Years	T	30	—
Steps per year	n	12	Monthly
Trials	—	10,000	Independent GBM paths
Trigger thresholds	δ	5 %, 10 %, 15 %	Symmetric
Friction cost	c	10 bps	Nominal transaction cost

D.5 Conclusion

The Monte Carlo evidence supports the theoretical claim that an episodic, trigger-based financing rule can offset volatility drag in expectation. A ±10 % deviation threshold provides near-optimal restoration of the expected compounding path with roughly quarterly frequency under frictionless conditions. Introducing spreads, taxes, or leverage caps would bound the effect but not reverse its direction. Together with the analytical sections of this paper, these results establish a coherent empirical foundation for Spiral Theory's central proposition that disciplined episodic borrowing stabilizes wealth accumulation through time.

D.6 Invitation to critique, replication and interactive simulation

An interactive simulator implementing the episodic-borrowing rule is available at
https://SpiralTheory.ai

Users can set expected return, volatility (or variance), risk-free rate, borrowing costs, leverage caps, and cash-flow schedules, and observe induced distributions of geometric growth, time-to-target, and drawdowns. We characterize results from the simulator as computational evidence consistent with the model; they are not a substitute for the identity itself.

We welcome critique, replication attempts, adversarial tests, and contrary evidence. We will maintain a registry of submissions; those that falsify particular claims will be documented and addressed in updates to the companion evidence paper. Please send evidence to: https://www.tja.global/contact

Endnote I. Relation to existing approaches

Kelly/log-utility. Kelly maximizes expected log-wealth by choosing a constant exposure and continuously rebalancing. Its objective is optimal growth given a utility criterion. Spiral Theory's objective is different: it offsets the arithmetic–geometric gap for a given risky sleeve so that the long-run geometric return equals the sleeve's arithmetic expectation. The policy is ex post (act after deviations), not continuous ex ante rebalancing. Under the canonical assumptions, the identity is about eliminating a compounding penalty, not about utility maximization; the two programs can coincide in special cases but are not equivalent.

Volatility harvesting via constant-weight rebalancing. The rebalancing premium arises from diversification and convexity across imperfectly correlated assets; many derivations assume some degree of mean reversion or rely on cross-asset dispersion. Spiral Theory does not require multiple assets or mean reversion; it operates on a single risky sleeve and the risk-free asset, with the borrowing option offsetting volatility drag in expectation under i.i.d. Gaussian returns. Any premium from multi-asset rebalancing is orthogonal to the variance-offset channel here.

Volatility targeting/risk parity. Volatility targeting scales exposure ex ante in response to realized or forecast volatility to stabilize ex ante risk. Spiral Theory conditions on realized return shortfalls relative to expectation, not on volatility levels, and uses financing adjustments ex post. The two can be combined, but their mechanisms and state variables differ.

CPPI and floor-based methods. CPPI implements a floor by reducing risky exposure after losses (selling into drawdowns). Spiral Theory does the opposite on the financing margin—after losses it adds financed exposure to offset the compounding penalty—subject to constraints. CPPI's objective is capital preservation relative to a floor; Spiral Theory's is restoration of long-run compounding efficiency.

Market timing/forecasting. Spiral Theory makes no forecasts. Actions are mechanically triggered by realized deviations from the expected path; the borrowing function lags the drag. Any realized uplift under the assumptions is therefore not a timing premium.

Dollar-cost averaging (DCA). DCA adds more dollars after price declines via contributions; Spiral Theory generalizes the same "buy after adverse realization" logic to financing rather than to external cash flows. Without borrowable liquidity, DCA is the nearest analogue; with borrowable liquidity, episodic financing symmetrizes the mechanism even in the absence of new savings.

Scope of claim. The identity established here is conditional: under the canonical MPT assumptions, episodic financing offsets volatility drag in expectation and restores the CAL's realized slope over time for the risky sleeve. It does not assert dominance over Kelly, rebalancing premia, volatility targeting, or CPPI in all environments; those are distinct objectives and mechanisms. In practice, frictions, taxes, borrowing spreads, leverage caps, collateral, and tail behavior bound the achievable uplift and determine which combination of methods is welfare-improving.

Relation to Merton Share

Relation to Merton's optimal risky share. In Merton's framework with stable investment opportunities and standard risk aversion, the investor holds a fixed fraction of wealth in the risky portfolio; that fraction rises with the expected excess return, falls with return variability, and falls with the investor's risk aversion. Spiral Theory leaves that choice untouched. Conditional on whatever risky fraction the investor or planner selects—including the Merton fraction—the episodic financing rule acts only on the financing margin, after the fact, to offset the compounding penalty that otherwise makes realized growth fall short of the sleeve's arithmetic expectation. It does not change the portfolio's composition or rely on forecasts; it borrows after adverse realizations and repays after favorable ones so that, on average, exposure stays close to the chosen fraction while long-run compounding efficiency is restored. In practice, borrowing costs and leverage limits bound the overlay; a utility-consistent implementation sets those bounds so the average exposure remains near the chosen share, using episodic adjustments solely to neutralize volatility drag rather than to seek additional risk premia.

Merton chooses how much risky asset to hold; Spiral Theory shows how to finance that choice through time so that realized compounding matches the model's expectation

Endnote II. Why this is the missing bridge to Modigliani–Miller

MPT is an asset-selection model: choose a risky sleeve and scale it along a straight capital-allocation line; financing sits off-stage. Modigliani–Miller is a financing model: in frictionless markets, leverage does not change value; in practice, firms treat liquidity as an option and manage balance sheets episodically. Portfolio thinking absorbed the first and largely ignored the second, leaving a gap between investment philosophy and the CFO's balance-sheet practice.

Spiral Theory closes that gap without disturbing either foundation. It keeps MPT's point-in-time results intact—no new frontier, no new Sharpe—and imports the corporate-finance idea that matters through time: financing flexibility is an option. A rules-based policy—borrow after adverse realizations, repay after favorable ones—neutralizes the compounding penalty that MPT leaves unpriced. Debt is not added to chase a steeper CAL; it is used to keep realized compounding aligned with the slope MPT already implies.

This reframes household and endowment choices excluded from MPT. A fixed-rate, non-callable mortgage paired with liquid reserves is not "cash at a negative spread"; it is an option to shift the quantity of money when it matters. Owning outright extinguishes that option; term financing plus cash preserves it—the same logic that governs corporate credit lines, operating cash, and maturity policy. Spiral Theory treats financing as a third control variable (alongside asset mix and spending) and shows how episodic, rules-based debt can restore compounding efficiency under the same assumptions that support MM separation and mean–variance analysis.

The claim is conditional and testable. Where financing is costly, callable, or tightly constrained, the option's value is bounded and the uplift attenuates; where funding is reliable and cheap relative to expected returns, the corporate-finance logic carries over cleanly. The models then interlock: MPT chooses the sleeve; Modigliani–Miller explains point-in-time irrelevance of fixed leverage; Spiral Theory supplies the over-time financing policy that preserves the sleeve's expected compounding in realized outcomes.

From the author

This paper aims to meet academic standards most of the way: assumptions are explicit, the result is labeled as an identity in expectation, and empirical questions are separated from the model. I ask in turn that critics meet the last mile of practice. Real households cannot rebalance against a house; many endowments and pensions lack a financing policy. Liquidity has option value—CFOs have priced it for decades. In standard diagrams the quantity of money sits off-stage; in real planning it determines whether one can act when it matters. Spiral Theory puts that variable on stage and gives it a disciplined role: borrow after adverse realizations, repay after favorable ones, within bounds. It does not change the risky sleeve or claim forecasting skill. Under the assumptions, the result is an identity; in practice, costs, caps, taxes, and tails bound the gain. If one believes the world is riskier, the mechanism becomes more valuable to do correctly—and more dangerous to overdo.

To make scrutiny easy, I have released tools so others can simulate, critique, and attempt to break the approach. I also disclose an intent to commercialize these ideas through AI-enabled, outcome-based planning and fee-for-service advice. The claim here is not certainty: it is a clean identity under clean assumptions, a rule that turns an obstacle—volatility drag—into a method, and an open invitation to test the mechanism against one's priors.

In the end the bridge is straightforward. Modern Portfolio Theory chooses the sleeve; Modigliani–Miller explains why fixed capital structure does not change that sleeve's value at a point in time. Spiral Theory supplies the over-time financing policy that keeps realized compounding aligned with the slope the model already implies—on the balance sheet, with the quantity of money in plain view—so more of us can hold two states at once: risky now, better over time.

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Plain-English caution

Nothing here is investment advice. Borrowing introduces real risks and constraints; outcomes depend on costs, collateral, taxes, and tail behavior. Test before use, and fit the policy to your balance sheet, not the other way around.

